Midterm Exam Solutions

6 March 2013

You have up to 1 hour, 45 minutes. You may use a simple calculator, but no text book or notes.

1. For each statement below, fill in the blank with the *best* term from the following list. Some terms might be used more than once; some might not be used at all.

   • algorithm • ASCII • bit • Boolean • byte • hexadecimal • pixel • pseudo-code • Python • unicode

   (a) A(n) byte is a binary value that can have 256 possible states.

   (b) A(n) algorithm is a finite sequence of effectively-computable instructions intended to produce a desired result.

   (c) Named after a 19th-century mathematician, a(n) Boolean is a value that is either true or false.

   (d) ASCII is a 7-bit encoding of characters commonly used in American English.

   (e) A(n) pixel is the smallest element of a solid color in a digital image.

2. Convert the following decimal (base 10) integers into 5-bit signed two's complement binary numbers.

   (a) $+14$ = 0 1 1 1 0

   (b) -13 = 1 0 0 1 1

   (c) $+3$ = 0 0 0 1 1

   (d) -3 = 1 1 1 0 1

   (e) -1 = 1 1 1 1 1

3. Add the following pairs of 6-bit signed (two's complement) binary numbers. Your answers must be in binary, but you may wish to check your work by converting to decimal. Remember, values can be negative!

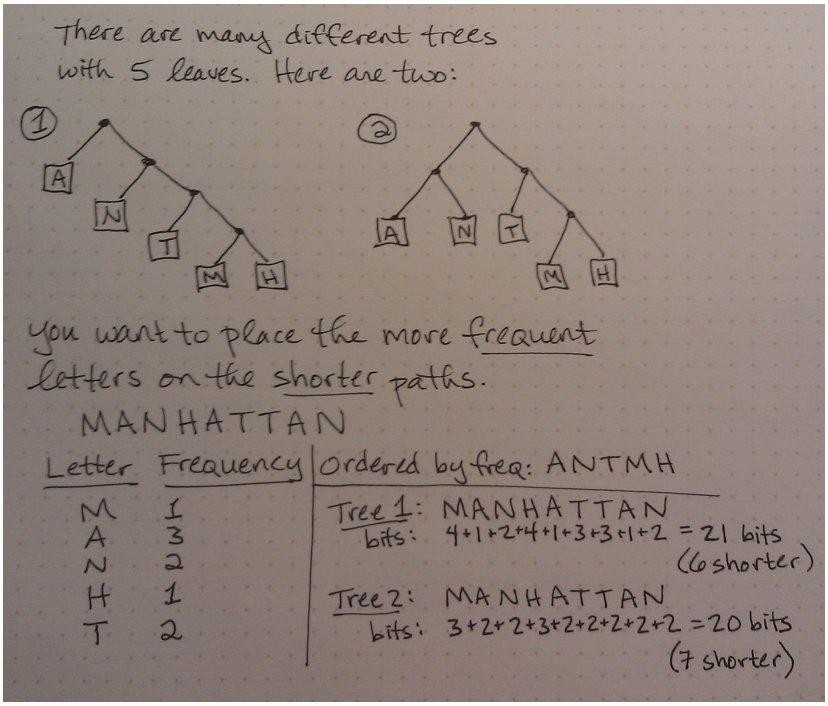
```
1 1 1 1 1              1 1 1 1 1 1              1 1 1 1 1
  1 0 1 0 1 0 = -22       0 0 1 1 1 1 = +15       0 1 1 0 1 1 = +27
+ 1 1 0 1 1 0 = -10     + 1 1 0 0 1 1 = -13     + 0 0 1 1 1 1 = +15
--------------         --------------           --------------
  1 0 0 0 0 0 = -32       0 0 0 0 1 0 = +2        1 0 1 0 1 0 = -22
```

4. Suppose we want to design encodings just for the five letters `A`, `H`, `M`, `N`, and `T`.

   (a) How many bits would we need to represent each letter in a **fixed-width** encoding? 3

   (b) Using the fixed-width encoding in the previous question, how many bits would we need to represent the nine-letter word `MANHATTAN`? 27

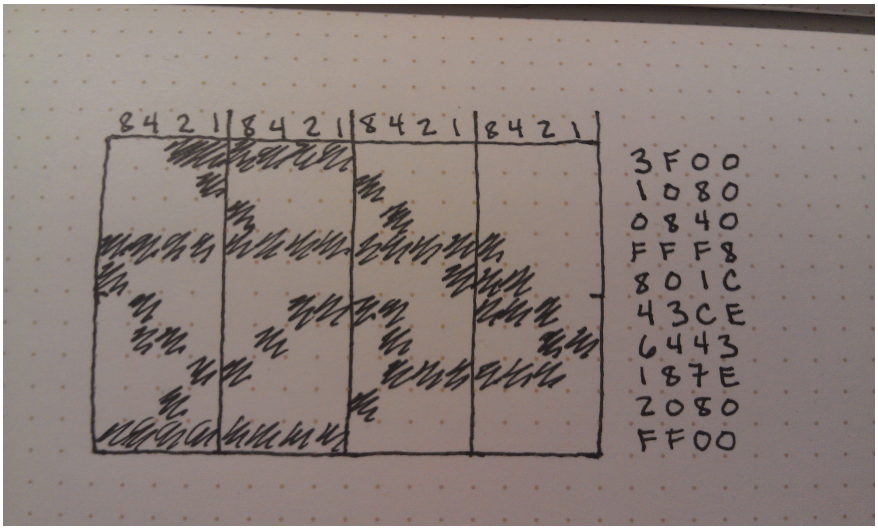
   (c) Draw a tree to represent a **variable-width** encoding of these five letters. Use your tree to encode the word `MANHATTAN`. How many bits did you need? _ How many bits did you *save,* compared to the fixed-width encoding? _



5. Create a truth table to show the value of $(X{\cdot}Y)' + X$ for all possible inputs of X and Y.

X	Y	X·Y	(X·Y)'	(X·Y)'+X
0	0	0	1	1
0	1	0	1	1
1	0	0	1	1
1	1	1	0	1

6. Encode the following 16×10-pixel graphic of a space ship into hexadecimal notation, using 1 bit per pixel.



7. What is the output of the following algorithm? Remember to indicate clearly what is *output* and what is scratch work.

```
1. Set N to 1
2. Set K to 1
3. If K > 4 then output N and stop.
4. Set N to N * K
5. Set K to K + 1
6. Go back to step 3.
```

```
N: ~~1~~ ~~1~~ ~~2~~ ~~6~~ 24
K: ~~1~~ ~~2~~ ~~3~~ ~~4~~ 5
Output: 24
```

8. What is the output of the following algorithm?

```
1. Set P to 18
2. If P = 0 then stop
3. If P is even, then output 0 and set P to P/2
   Otherwise output 1 and set P to (P-1)/2
4. Go back to step 2.
```

```
P: ~~18~~ ~~9~~ ~~4~~ ~~2~~ ~~1~~ 0
Output: 0 1 0 0 1
```

9. What is the output of the following Python program?

```
four = 4
six = four + 2
print "six is six"
six = six - 3
print six+1
four = four * four
print four+4
print "five * four"
```

```
six is six
4
20
five * four
```