Assignment 2 – text compression

due in class on Tue 18 Feb (40 points)

This assignment is an activity for groups of three. We'll work on it in class on Wed 5 Feb, and then your group must submit **one** set of responses to the six questions before the deadline.

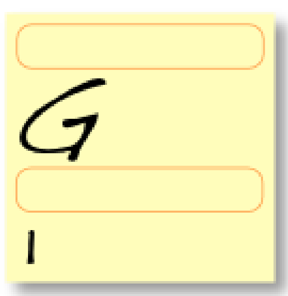
Introduction

In this activity, we will investigate the Huffman algorithm for text compression. You've already seen one example of a Huffman encoding, represented by the strange-looking tree on the handout labeled "variable-bit Huffman encoding."

You will follow the Huffman algorithm and create a tree of your own, based on the character frequencies of a message that I provide.

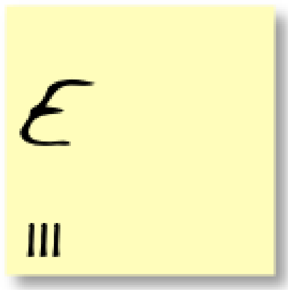
Phase 1: count letter frequency

Start with a stack of blank sticky notes and the message you were given. We're going to consider each of the characters in your message, in order. Suppose the first character is a **G.** We would write the **G** on a sticky note – roughly at the center left – and also begin a tally in the lower left corner. Leave some space above and below the character, as shown:

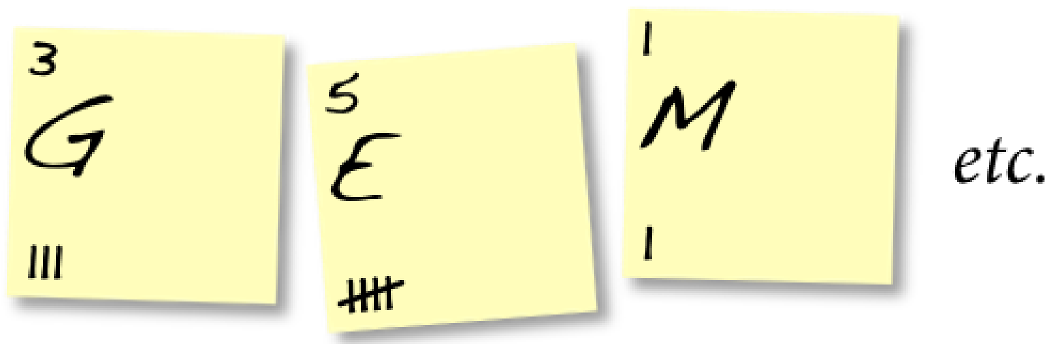


Move on to the next character in your message. Assuming it is a different character, make a new sticky for that one.

When you encounter a character that you've seen before, do **not** create a new note, but instead update the tally on the existing note containing that character. In this example, we've just seen the character **E** for the third time:



Continue doing this for the entire length of your message. You will now have a count of the frequencies of each character. Write the frequency in conventional (base ten) notation in the upper left. Here's a small sample:



In the next section, we will process these characters in order from lowest frequency to highest. So you may want to take a moment now to arrange them in roughly that order on your desktop.

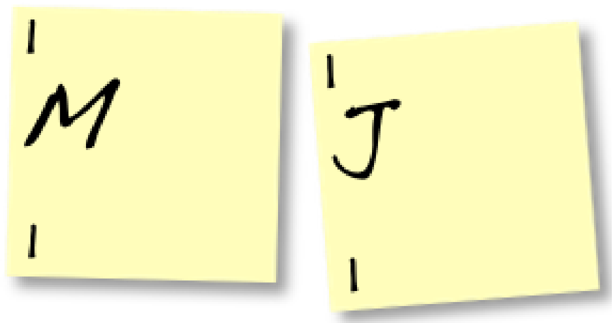
Question 1: How many distinct characters did your message contain?

Question 2: If we were using a fixed-width encoding, how many bits would you need to represent just those characters?

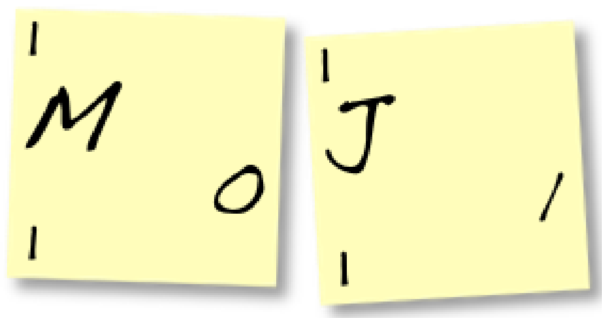
Question 3: What is the most frequent character in your message, and how many times did it appear?

Phase 2: merge tiles

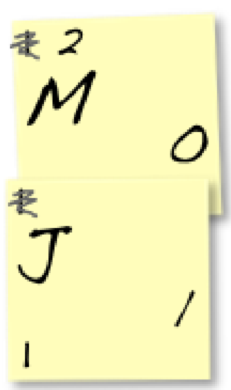
The algorithm continues by repeatedly merging sticky notes, as described here. Start by choosing two notes with the lowest frequencies. Probably you had several characters with a frequency of one, so you can just choose two of them arbitrarily. Place them side by side in your work area:



In the section beneath the character and **starting from the right,** write a zero on the left note and a one on the right note:



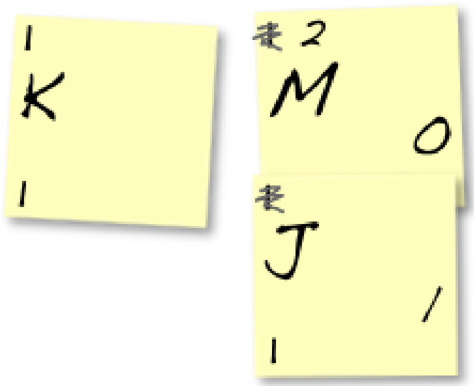
Then, stick the right one onto the bottom of the left one. Cross out the frequencies and replace the top one with their **sum** – in this case, 1+1 is 2:



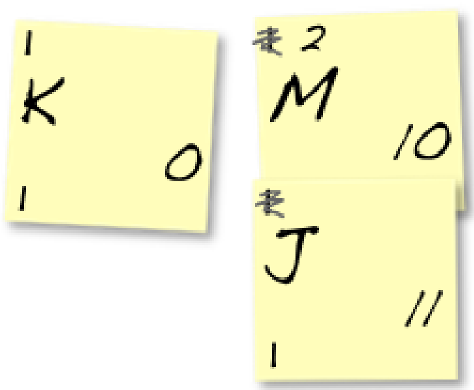
Now you will treat this 'merged' note as if it were a single one, so place it somewhere among other characters with frequency=2.

Continue merging together your lowest-frequency letters like this. It's okay to pair a frequency=1 with a frequency=2 if it's the last frequency=1 remaining – then the merged frequency would be 3.

Before long, you'll have to merge notes that themselves are already merged. In the example below, we paired the last frequency=1 character (**K**) with a group (**MJ**) that has frequency=2:



As before, we write a zero on the left note. And we write ones on **all** of the right notes… to the left of whatever code is already there:



Again, stick the right note onto the bottom of the left one. Cross out the frequencies and replace the top one with their **sum** – in this case, 1+2 is 3.

Continue merging notes using this technique until every character in your message is merged into one big note. Then you will have a distinct binary encoding underneath each character. Probably you should take a photo of your encoding, and/or write down the bits produced for each character elsewhere.

Question 4: How many bits are used to represent the most frequent character in your message?

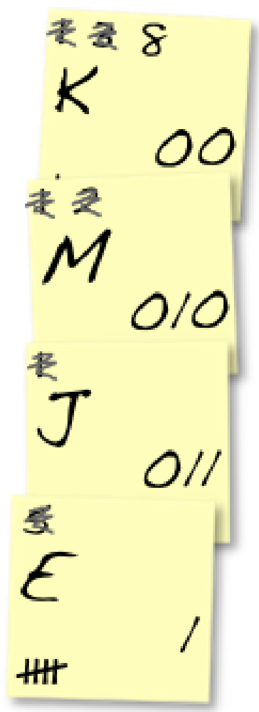
Question 5: What is the most number of bits used to encode any character in your message?

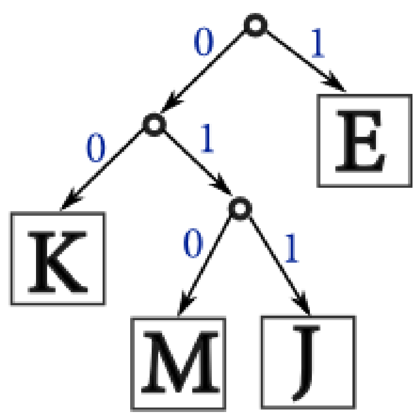
Question 6: Use the character encodings you produced to encode the entire message you were given. How many bits are used, in total?

Visualize encoding as a tree

As in the handout on variable-bit Huffman encoding, the character encodings you produced should fit nicely into a binary tree. I'll do a small example below. Our algorithm has produced the encodings `00` for **K,** `010` for **M,** `011` for **J,** and `1` for **E.**

We interpret a `0` as choosing the **left** path in a binary tree, and `1` as the **right** path. So to get to the **K** from the root we would go left, twice. For the **E,** we go right just once. The **M** and **J** both have the prefix `01`, so they sit at a "sub-tree" reached by going left then right.





Task: Draw the entire tree corresponding to the character encoding you produced using the Huffman algorithm.