

Figure 1:

## Boolean logic

### Boolean algebra and logic gates

In the 1840s, English mathematician George Boole developed an algebra (a set of operators and laws) for variables that can have just two states – **true** and **false**. Thus, a Boolean value is equivalent to one bit:

False = 0 = off  
True = 1 = on

The operators defined by Boole are pervasive throughout all of computing. You may have encountered them in doing library or other database searches. The ones we'll consider are:

The table illustrates both the algebraic notation and the **circuit diagram** notation. The elements of circuit diagrams are called **gates**, as in “AND gate” or “XOR gate.”

The behavior of these operators can be defined by **truth tables**:

A	B	$A \cdot B$	$A+B$	$A'$	$A \oplus B$
0	0	0	0	1	0
0	1	0	1	1	1
1	0	0	1	0	1
1	1	1	1	0	0

### Combinational circuits

We combine the gates into **combinational circuits** to achieve various effects. For example, the algebraic expression  $X = A \cdot B + A \cdot C$  corresponds precisely to the fol-

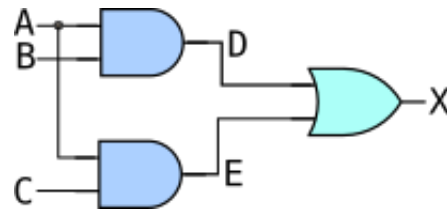


Figure 2:

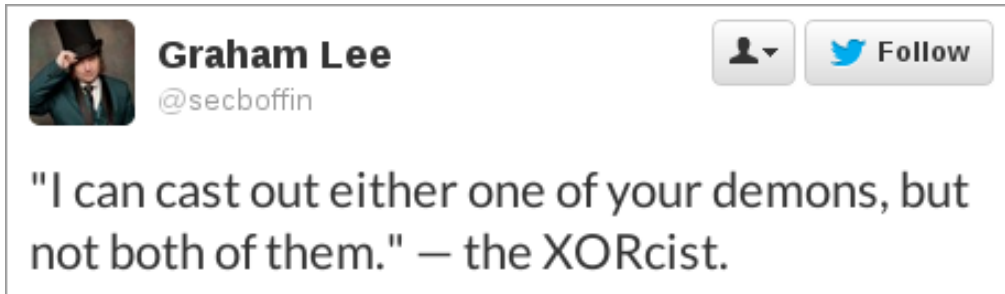


Figure 3: @secboffin on Twitter

following circuit diagram:

and we can discover its effect by completing the truth table:

A	B	C	$D=A \cdot B$	$E=A \cdot C$	$X=D+E$
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1

**Exercise:** Try drawing the circuits and the truth tables for  $X=(A \cdot B)'$  and for  $X=A'+B'$ . They should produce the same result for the inputs A and B. This is one of **DeMorgan's Laws**.

## Sequential circuits

We'll just look at the S-R (NAND) latch.

- [Flip Flops, Latches & Memory Details](#) [8:53] from Computerphile

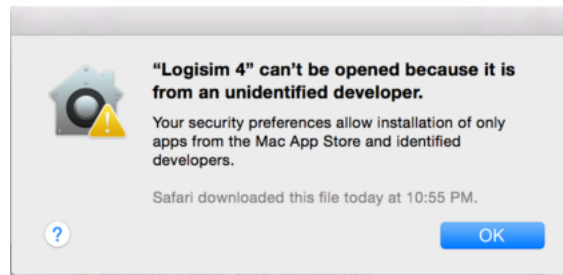


Figure 4:

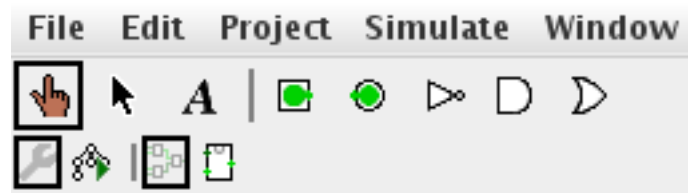


Figure 5:

## Logisim software

This section refers to a program called [Logisim](#), which should run on any platform with a Java Runtime Environment. If you get the “unidentified developer” error on a Mac, go into **System Preferences** » **Security** and look for the button that says **Open Anyway**.

Once you open Logisim, there are a few tools you should familiarize yourself with.

The ‘hand’ tool (leftmost on the toolbar) allows you to **turn inputs on and off**. The ‘arrow’ tool (next to it) allows you to place components onto the grid, move them around, and wire them together.

In the side-bar, the main components we’ll be using are in the **Gates** section, but there’s also the **Pin** (under Wiring) and the **LED** (under Input/Output).

When you have a component selected, its properties appear in the lower left of the screen. You can use these to create a label for your pins and LEDs.

- The [3-bit adder circuit](#) I did in class.

## Further exploration

- Video: [Building a half-adder using dominoes](#), with Matt Parker and Numberphile
- Video: [The big domino adder](#), demonstrated at the Manchester Science Festival, UK

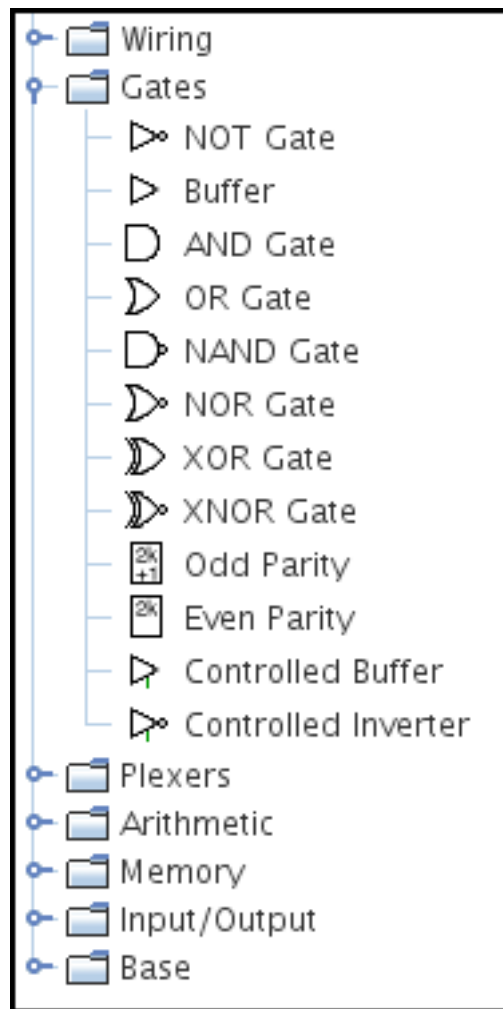


Figure 6:

Selection: LED	
Facing	North
On Color	#f00000
Off Color	#404040
Active On High?	Yes
Label	Carry
Label Location	East
Label Font	SansSerif Plain 12
Label Color	#000000

Figure 7:



Figure 8: [@Cmdr\\_Hadfield](#) on Twitter



Figure 9: