# Boolean logic



## Contents

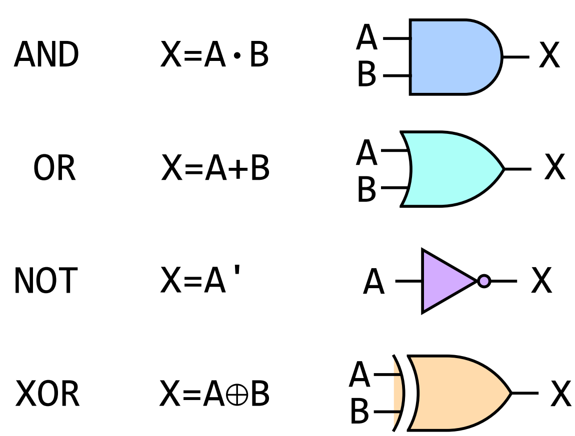


## 1. Boolean algebra and logic gates

In the 1840s, English mathematician George Boole developed an algebra (a set of operators and laws) for variables that can have just two states – **true** and **false.** Thus, a Boolean value is equivalent to one bit:

| | | |
|---|---|---|
| False | 0 | off |
| True | 1 | on |

The operators defined by Boole are pervasive throughout all of computing. You may have encountered them in doing library or other database searches. The ones we'll consider are:




The figure illustrates both the algebraic notation and the **circuit diagram** notation. The elements of circuit diagrams are called **gates,** as in "AND gate" or "XOR gate." The "XOR" (⊕) operator is named for "*exclusive* OR."

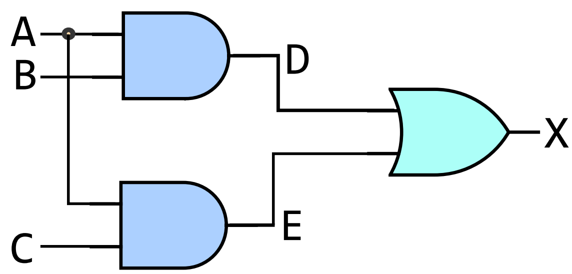
The behavior of these operators can be defined by **truth tables:**

| A | B | A·B | A+B | A′ | A⊕B |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 | 0 |

The first two columns indicate the values of the two variables, A and B. There are four rows because two variables can take on four different values ($2^2 = 4$). If there were three variables, there would need to be $2^3 = 8$ rows.

## 2. Combinational circuits

We combine the gates into **combinational circuits** to achieve various effects. For example, the algebraic expression `X = A·B + A·C` corresponds precisely to the following circuit diagram:



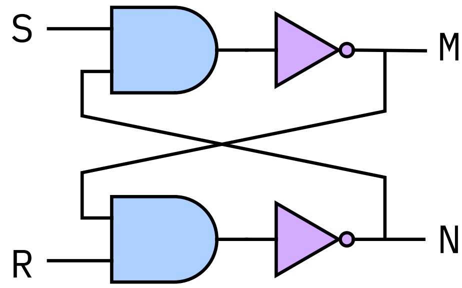

and we can discover its results by completing the truth table:

| A | B | C | D=A·B | E=A·C | X=D+E |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

**Exercise:** Try drawing the circuits and the truth tables for X=(A·B)′ and for Y=A′+B′. They should produce the same result for all inputs A and B. This is one of **DeMorgan's Laws.**

## 3. Sequential circuits

We'll just look at the S-R (NAND) latch.




This is a **sequential** circuit, rather than combinational. That means it contains *cycles*. One way to make sense of a cycle is to think in terms of the values of wires *from one moment to the next*. You can subscript each variable with the time of interest: $S_t$ vs $S_{t+1}$, etc.

```
M[t+1] = (S · N[t])'
N[t+1] = (M[t] · R)'
```

| S | R | M[t] | N[t] | M[t+1] | N[t+1] |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 |

- Video: Flip Flops, Latches, & Memory[1] from Computerphile [8m53s]

[1] youtu.be/-Ecf71b4aZ0

## 4. Logisim software

This section refers to a program called Logisim[2], which should run on any platform with a Java Runtime Environment.

[2] www.cburch.com/logisim/download.html

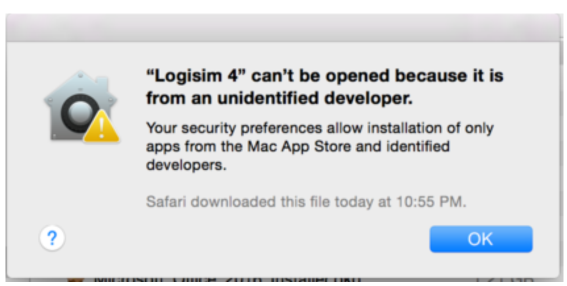
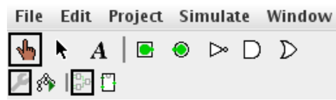


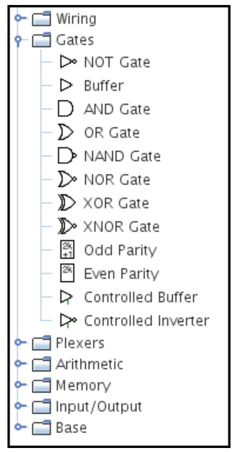

Figure 1: On a Mac, if you see the "unidentified developer" error, go **into System Preferences » Security** and look for the button that says **Open Anyway.**

Once you open Logisim, there are a few tools you should familiarize yourself with. The **hand tool** (leftmost on the toolbar) allows you to *turn inputs on and off.* The **arrow**

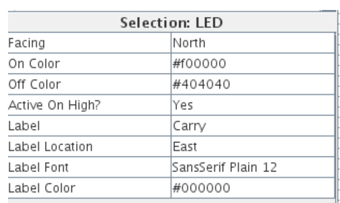
**tool** (next to it) allows you to place components onto the grid, move them around, and wire them together.




In the side-bar, the main components we'll be using are in the **Gates** section, but there's also the **Pin** (under Wiring) and the **LED** (under Input/Output).




When you have a component selected, its properties appear in the lower left of the screen. You can use these to create a label for your pins and LEDs.



| Selection: LED | |
|---|---|
| Facing | North |
| On Color | #f00000 |
| Off Color | #404040 |
| Active On High? | Yes |
| Label | Carry |
| Label Location | East |
| Label Font | SansSerif Plain 12 |
| Label Color | #000000 |

Here is the 3-bit adder circuit I did in class. If that file doesn't open automatically in Logisim, you can start Logisim *first* and then use **File » Open**.

- Video: Logisim tutorial[3] from ENGRTUTOR [7m47s]

## 5. Further exploration

- Video: Building a half-adder using dominoes[4], with Matt Parker on Numberphile [18m30s]
- Video: The big domino adder[5], demonstrated at the Manchester Science Festival, UK [22m26s]

[3] youtu.be/ATPqpFMlVdw

[4] youtu.be/lNuPy-r1GuQ

[5] youtu.be/OpLU__bhu2w

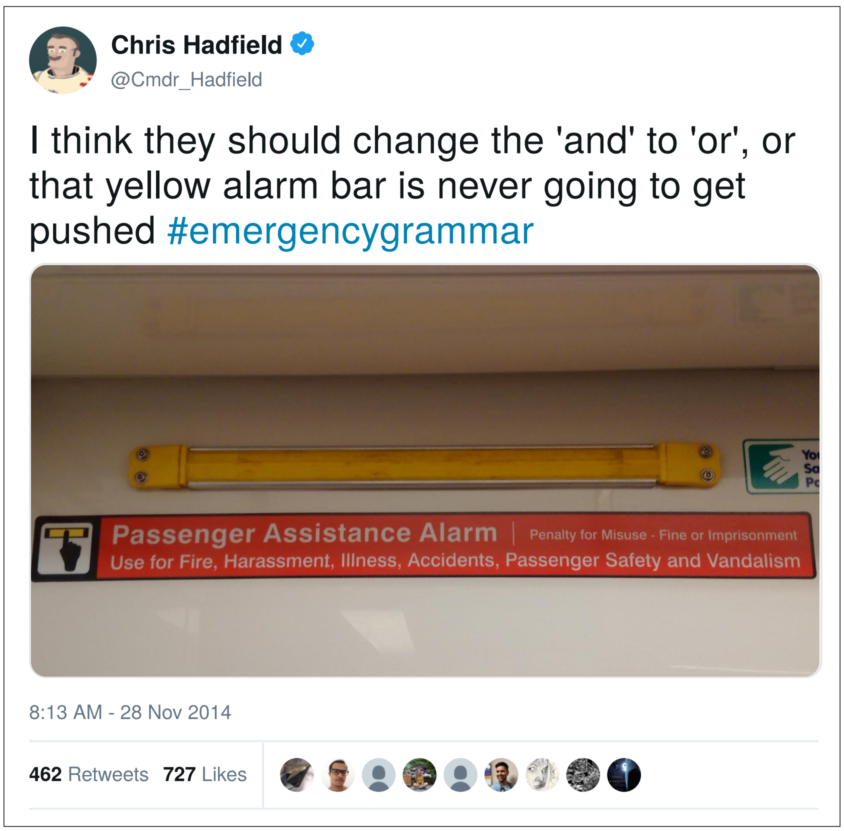



Figure 2: `@Cmdr_Hadfield` on Twitter