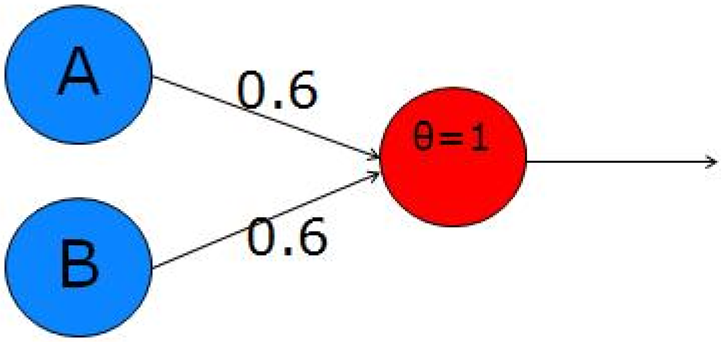


Figure 1: Sample Perceptron from this overview

Perceptrons

These are rough notes from our meeting on Thursday 19 January.

We (I) decided to pursue a project in deep reinforcement learning, perhaps with application to game-playing. I referenced these projects and papers:

- OpenAI Universe
- Metalforge Crossfire
- Human-level control through deep reinforcement learning, *Nature* **518**, 26 February 2015

Basics

To begin exploring this area, we started with the simplest artificial neural networks, known as **perceptrons.** This formulation began with McCulloch and Pitts in **1943!** (I was off by a few decades in class.)

We have a graph in which both nodes (circles) and edges (lines) are assigned real numbers in some range, typically 0..1 or -1..+1, but other ranges can be used too. Numbers assigned to nodes are called **activation levels,** and numbers assigned to edges are called **weights.**

In this perceptron, let's call the two input nodes (left layer) A and B, and the output node is C. The upper edge weight will be w_a and the lower edge weight is w_b. The main calculation is just a weighted average: $C = A{\cdot}w_a + B{\cdot}w_b$.

But then we also apply a function to the result, to adjust its range and kind of "snap" it into a positive or negative result (activated or inactive). This function can be a simple "step" with a given threshold t, such as $t = 0.5$ or $t = 1.0$:

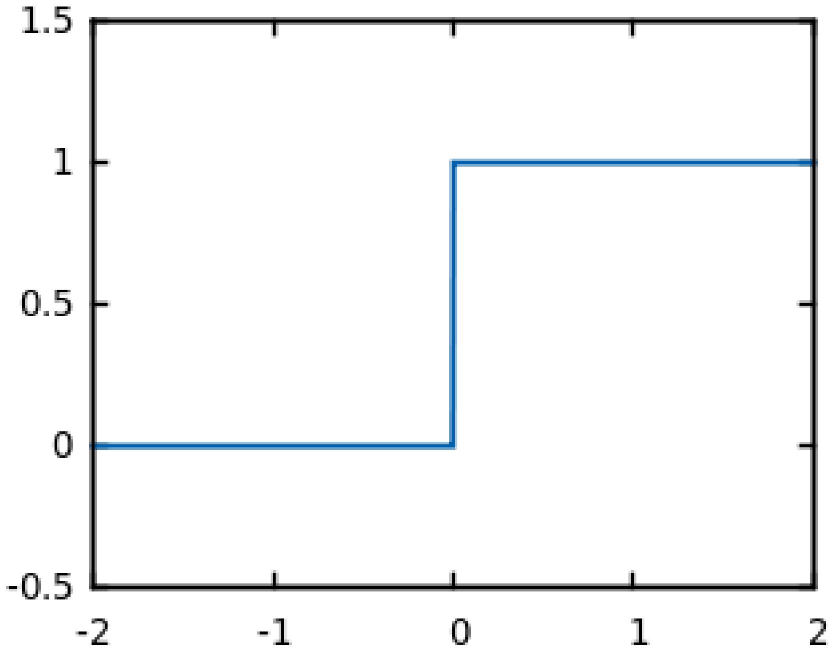


Figure 2: Step function, with threshold $t = 0$. [Source]

$$f(x) = \begin{cases} 0 & \text{if } x < t \\ 1 & \text{if } x \geq t \end{cases}$$

Later on, we may use a more sophisticated function that smooths out the discontinuity at the threshold, like this one, called the Sigmoid function:

$$f(x) = \frac{1}{1 + e^{-x}}$$

It's interesting to see if we can make perceptrons emulate the Boolean logic operators, like AND, OR, NOT, XOR. The perceptron above, with weights $w_a = 0.6$ and $w_b = 0.6$ implements OR:

```
A B  C = f(A*Wa + B*Wb)
0 0  f(0*0.6 + 0*0.6) = f(0)   = 0
0 1  f(0*0.6 + 1*0.6) = f(0.6) = 1  (because 0.6 > t)
1 0  f(1*0.6 + 0*0.6) = f(0.6) = 1
1 1  f(1*0.6 + 1*0.6) = f(1.2) = 1
```

Here we're using the step function with threshold $t = 0.5$.

We can implement Boolean AND with $w_a = 0.4$ and $w_b = 0.4$:

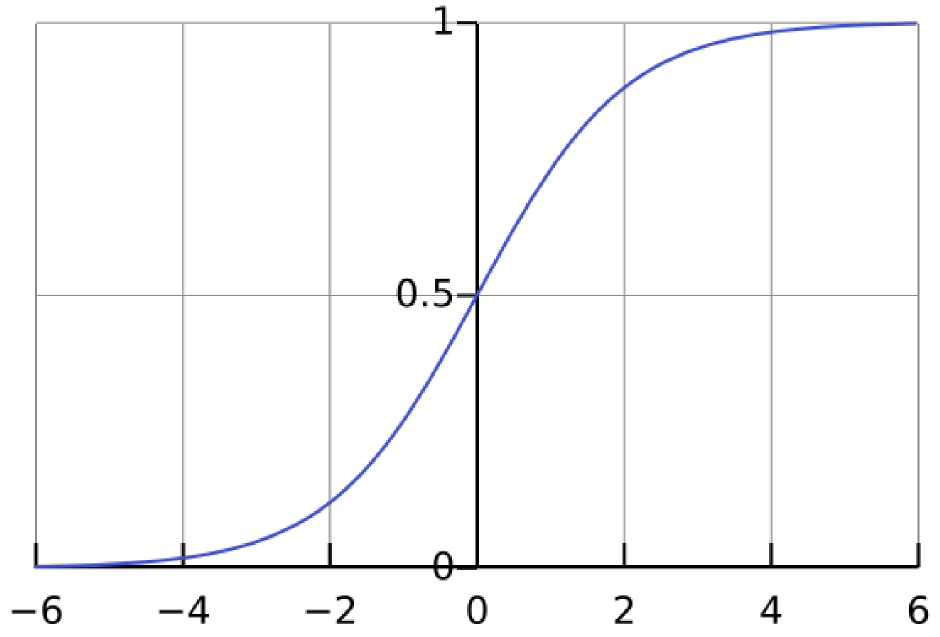


Figure 3: The "logistic" sigmoid (formula above), centered on $x = 0$. [Wikimedia]

```
A B  C = f(A*Wa + B*Wb)
0 0  f(0*0.4 + 0*0.4) = f(0)   = 0
0 1  f(0*0.4 + 1*0.4) = f(0.4) = 0  (because 0.4 < t)
1 0  f(1*0.4 + 0*0.4) = f(0.4) = 0
1 1  f(1*0.4 + 1*0.4) = f(0.8) = 1  (because 0.8 > t)
```

XOR

A problem arises with the XOR function. Minsky and Papert showed that this simple perceptron model cannot encode XOR. (And their influence set back research into artificial neural networks for a decade or more!) A perceptron can model (and learn) any function that is **linearly separable,** but XOR is not.

The trick to making this model more powerful is to add a "hidden" layer between the input nodes and the output node. Then you fully-connect the nodes of the input layer with those in the hidden layer. That produces a graph with five nodes and six edges:

We struggled a bit with getting this to work, but Priya figured it out! The solution is to set the threshold of the step function to 1. Below is her work:

In retrospect, I now understand the use of the θ (theta) label in these diagrams: it indicates the threshold value to use to assign a value to the labeled node. So $= 1$ was staring us in the face!

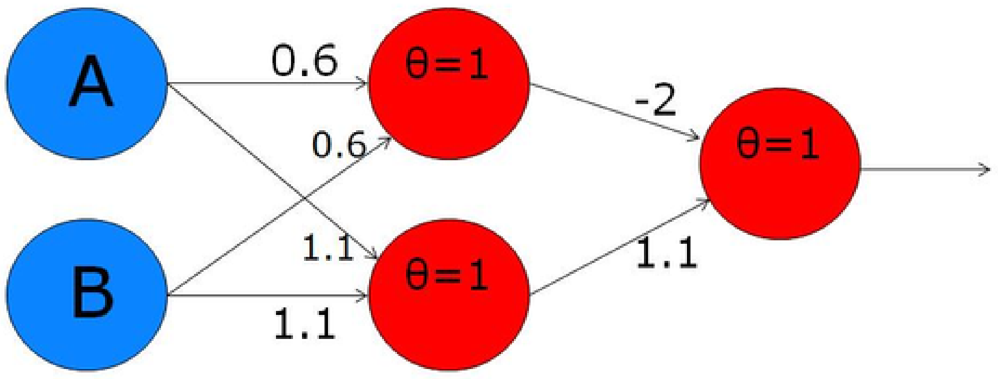


Figure 4: XOR implementation with hidden layer

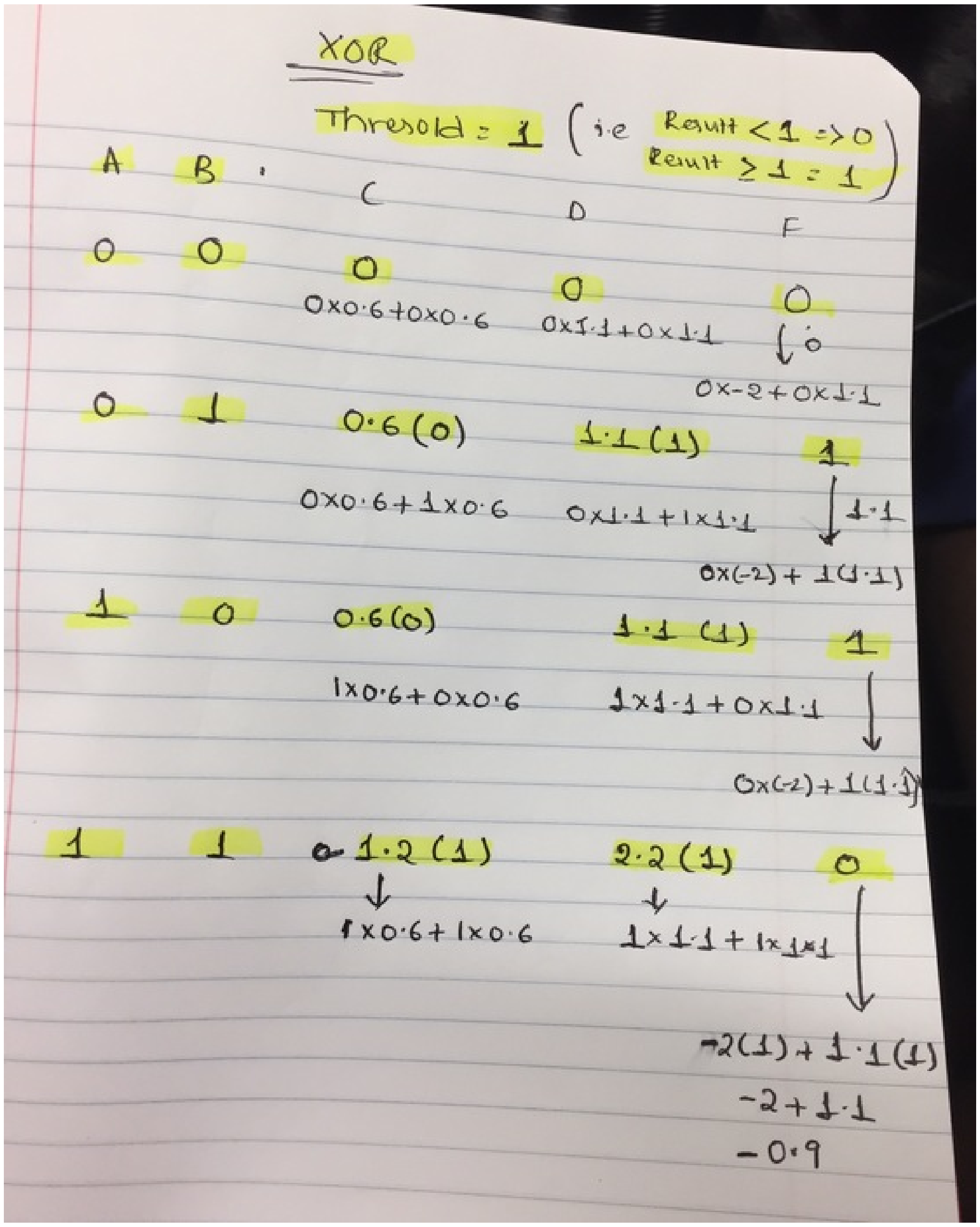
Code

Below is a sketch of a small C++ program to implement simple fixed-size perceptrons (three nodes, two edges). It includes a `main` program that – once the functions are working correctly – will print the truth tables for AND, OR, and NAND! See the sections marked “TODO”. Here is the expected output:

```
AND:
0 0: 0
0 1: 0
1 0: 0
1 1: 1
OR:
0 0: 0
0 1: 1
1 0: 1
1 1: 1
NAND:
0 0: 1
0 1: 1
1 0: 1
1 1: 0
```

```
// Simple perceptron logic gate implementation
#include <iostream>
using namespace std;

float step_function(float threshold, float value)
{
  // TODO
  return 0;
```



XOR

Thresold = 1 (i.e Result < 1 => 0
Result ≥ 1 = 1)

A	B	C	D	F
0	0	0 0x0·6+0x0·6	0 0x1·1+0x1·1	0 ↓ 0 0x-2+0x1·1
0	1	0·6 (0) 0x0·6+1x0·6	1·1 (1) 0x1·1+1x1·1	1 ↓ 1·1 0x(-2)+1(1·1)
1	0	0·6 (0) 1x0·6+0x0·6	1·1 (1) 1x1·1+0x1·1	1 ↓ 0x(-2)+1(1·1)
1	1	~~0~~ 1·2 (1) ↓ 1x0·6+1x0·6	2·2 (1) ↓ 1x1·1+1x1x1	0 ↓ -2(1)+1·1(1) -2+1·1 -0·9

Figure 5: Full resolution

```
}

float run_perceptron(float weightA, float weightB,
                     float valueA, float valueB,
                     float threshold)
{
  // TODO
  return 0;
}

void show_truth_table(float weightA, float weightB,
                      float threshold)
{
  // TODO: call run_perceptron 4 times,
  // feeding it all four values of A,B.
}

int main()
{
  cout << "AND:\n";
  show_truth_table(0.4, 0.4, 0.5);
  cout << "OR:\n";
  show_truth_table(0.6, 0.6, 0.5);

  // TODO: can you figure out two weights and a threshold
  // that will implement "NAND"? This is the opposite of AND.
  cout << "NAND:\n";
  show_truth_table(0,0,0);
  return 0;
}
```